Death by x and y – a mathematician's lament

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Dedicated to class 2.F of 2010-11

ABSTRACT
The Micro Project is a project assignment to small groups of learners, the outcome of which is an oral presentation (per group) attended by the other groups who supply constructive criticism. In mathematics, at least, it thus departs from the traditional book + lecture + exercise paradigm, which is known to stultify reflection, thus causing the mental 'death by x and y' mentioned in the title. Our collective failure to provide stimuli of mathematical reflection may form part of the cause of The Mathematics Problem, students' inadequate grasp of the mathematics they are assumed to master as they enter university. The Micro Project and experiences with its use in secondary education is described. Various challenges of carrying it over to tertiary education are listed along with suggestions as to how these can be (partially) met.

Keywords – action learning, micro project, mathematics teaching

I INTRODUCTION

It is possible to identify at least three separate, if interacting, sources of motivation leading to the musings of this paper:

- The Mathematics Problem – the gap between the mathematical skills university students are, often tacitly, assumed to possess and those they actually acquire during their years in primary and secondary school
- One sorely neglected thread in the web of reasons behind the Mathematics Problem, namely that we teach mathematics in the wrong way, or at least with an unfortunate bias
- The concept of action learning and one particular realization of it, called Micro Projects for want of a better name – which may to some extent help setting the pedagogical balance right as far as the teaching of mathematics is concerned

All three will be discussed in the following; but emphasis will be on the Micro Projects and the experiences the author had in developing and implementing them. By way of conclusion, the paper offers some reflections on the challenges meeting anyone who attempts to carry Micro Projects from their original setting in classroom teaching in the Gymnasium (the Danish version of High School) to university teaching.

II The Mathematics Problem

A very substantial literature exists on what has become known as *The Mathematics Problem*, and this is not the place to attempt a serious summary. An informal discussion of selected papers can be found in [12] (Danish) from which we can borrow the following, sufficient for our present purpose.

The Mathematics Problem, very tersely described, consists in the undeniable fact that students arrive at tertiary education – including university education – with an inadequate training in mathematics. This is

sometimes seen as a recent or local challenge and various simple explanations and quick remedies are offered; but in fact, a glance at the literature (as in [12]) will convince anyone with an open mind that:

- The problem is 30-40 years old
- The problem is international
- The problem is truly complex as must indeed its reasons be

Of the many individual causes identified by various writers one can mention

- Inadequate planning within and between primary and secondary school
- Inadequate focus on/handling of the Transition Problem, i.e. the fact that a young person going to university experiences a complete change of environment and in fact of life conditions
- Too little focus on new teaching methods, especially those making use of the computer
- Too much focus on new teaching methods, especially those making use of the computer (-!-)

At least a dozen such observations or claims are made in the serious literature on the problem – with several more in the popular press (many of them uninformed and groundless, but persistent).

It is clear, or so the author thinks, that none of these alleged causes can be held solely responsible for The Mathematics Problem but that they must be seen as interacting. But one possible such element is hardly ever discussed – although it is certainly brought up elsewhere, as the next section will attempt to demonstrate – namely that we may very well be teaching mathematics in the wrong way, not merely in terms of what we focus on, but in the entire conceptualization of mathematics as a subject to be learned.

III Do we teach mathematics with our eyes closed?

There is a wonderful observation ascribed to Pafnuty Chebyshev (-), se e.g. http://www-groups.dcs.st-and.ac.uk/~history/Quotations/Chebyshev.html

- To isolate mathematics from the practical demands of the sciences is to invite the sterility of a cow shut away from the bulls

A modern-day Chebyshev, Vladimir Arnol'd, voices a similar opinion in the opening lines of (Arnol'd, V. I. [3]):

- Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap

Arnol'd goes on to criticize the increasing preoccupation with abstraction. Although he is mainly concerned with mathematics at a higher level than that encountered by, say, first year engineering students, he does in fact describe his own - inimitable – experiences with teaching secondary level pupils.

Yet, although there are many wonderful exceptions, many – possibly most – text books used in first-year university mathematics courses contain endless arrays of sterile exercises and no practical examples, no attempts to connect the calculations to the reality known to, and inhabited by, the students.

Even the 1119-page *International Handbook of Mathematics Education* (Clements [8]) devotes only one chapter to mathematical modelling, and a rather dry one at that. In fairness, it should be added that it does

indeed have headlines such as Applicable Mathematics and Modelling for All and ensuing discussions, yet it seems – as far as the present author has managed to delve into it – to accept *status quo*.

So, there is a very real danger that a very large percentage of school children and 1st year students will be left with the impression that mathematics is "something to do with solving equations where you have to isolate *x*".

As for the gradual suppression of 'applied' mathematics in favour of the 'pure' version, the reader should consult (Maddy [15]). And to avoid painting too gloomy a picture: a collection of cases that actually provides some of what is here wished for can be found in e.g. (Sriraman et al [17])

But there is still more to this, as expressed by e.g. (Klamkin [13])

- I have thought for a long time that one of the most important goals of education is to get the students to 'think for themselves'. As I look over the American education scene, it seems that each year more and more material is being crowded into the curriculum. The net result being that most students hardly have any time to sit back and think out various problems for themselves. Consequently, most students will ***just parrot back*** the material from their texts or from their classroom notes

(Present author's emphasis, no offense to *psittacines* intended). A common observation along the same lines is voiced in (Bruckheimer and Gowar [7])

- The traditional emphasis is on technique as an end in itself; on gaining technical facility because that is what is wanted in examinations

An even deeper criticism is that we teach the isolated skills and leave it to the students to work out the big picture themselves – despite the recurrent emphasis of "the beauty of mathematics", *vide* Hardy.

It would be preposterous to suggest some simple remedy or other; but the next section describes what the author once did and hopes to do again, *mutatis mutandis*.

IV Micro-projects

The briefest of autobiographic sketches will provide the necessary background for the description to follow:

Author's note: After graduating as an M. Sc. Eng. in 1983, I was employed by the Technical University of Denmark (DTU) to write a text book on Linear Algebra. I next served as a research assistant at the Dept. of Mathematics, Univ. of Dundee 1984-1988, where during 1987 I also taught numerical solution of ordinary differential equations. Returning to DTU in 1988, I engaged in teaching of, and research in, Graphical Communication, Computer Graphics, programming, Numerical Analysis and various applications of mathematics. Leaving DTU in 2003, I was first employed at the Dept. of Education of the Danish Bankers' Association, where I served as a project manager responsible for an LMS while also developing an e-learning course in mathematics covering the Danish C-level (essentially 10th grade). After 4½ years at NeoConsult Aps., an IT company, as advisor and responsible for HRD, I returned to the world of education, serving as a teacher of mathematics and physics at Lyngby HTX (Danish "Teknisk Gymnasium") while also maintaining a secondary job as a mathematics tutor at DTU before returning full time to DTU primo 2014. End of note.

While at HTX, which educates 10-12th graders, emphasizing technical (and other practical) applications of most disciplines, the author felt compelled to develop modes of presenting mathematical material in a manner transcending the format of conventional exercises. One such was given the category name of *Micro Projects*. It was clearly inspired by earlier work on *action learning*, carried out at the Danish Bankers' Association and documented in (Hansen [11]), with roots further back in the teaching of Graphical Communication at DTU.

A Micro Project can in various ways be distinguished from conventional project assignments:

The class is subdivided into groups of (typically) 3-4 participants, each given a worksheet defining the task they must carry out. There is a common overarching subject, but each group works on a specific topic extracted from this subject. The goal is not a written report but an oral presentation. On the day of presentation, one group at a time is asked to explain its topic to the other groups, receiving immediate feedback in the form of (constructive) criticism from the audience. Typically, one other group is assigned the leading role as examiners. The teacher – the "grumpy corner" – will provide both the facilitation and a few extra comments in case *vox populi* has overlooked something.

A total of 24 Micro Projects in mathematics were planned, 8 per grade. (Similarly, 16 were planned for 10th and 11th grade physics). Of these, 17 reached the point where all ideas and material were collected, but only 6 were executed, as the author left HTX before the remainder could be carried through. (Of the Micro Projects in physics, 6 were executed, bringing the total of practical experiences with the form to 12, some repeated).

Many of the Micro Projects stayed close to the mathematical subject and were thus glorified exercises, albeit in the theory, rather than the calculations, pertaining to the subject. Yet, as it became clear that the pupils were easily capable of handling complex assignments, ambitions grew. Also, subjects first treated in project assignments were considered for a switch to this more liberal – and often very entertaining – format.

A full list of the subjects and their sub-topics can be made available to any interested reader. Here, two examples are listed in compact form and a further three discussed in more detail below:

Under the heading "Differentiation", the groups would have been asked to present the topics: 1) Curvature 2) Jounce 3) Taylor's theorem 4) Implicit functions (including the concept of a contour line) 5) The logarithmic derivative 6) Numerical differentiation 7) Splines and 8) Fractional calculus

(As for the latter, it was first made the topic of a project assignment and as such turned out quite successful despite its being rather advanced). This Micro Project was intended for a 12th grade class, but the author left HTX before the appropriate theory had been taught to this class.

In physics, under the heading "Pressure" the groups presented their work on: 1) The pressure under the feet of various animals 2) A pressure boiler 3) A lemon squeezer 4) The bubble 5) The paper ball 6) The diver's bell 7) The scroll pump 8) The siphon

This latter Micro Project was in fact implemented more than once and showed the ingenuity of the pupils when it came to the use of media such as short film clips, animations and various kinds of acting or role playing

Among the mathematical Micro Projects, we can take a closer look at: the first attempt; a failure; and a success:

The very first Micro Project was assigned to class 2.F of 2010-11 shortly after their first introduction to integral calculus. 2.F, a mathematics-biochemistry class, was small – hence only four sub-topics were needed – and with a wonderful social coherence that made it safe to try out this experiment.

The four topics were: 1) The areas of circular and elliptic annuli 2) Integration of Chebyshev polynomials 3) Various aspects of partial integration as exemplified by antiderivatives of $x \cdot \cos(x)$ and related functions; and 4) The concept of an integral with a variable limit

Altogether, the experiment was a success, although the pupils indicated quite clearly that this way of working was new to them. A fair amount of guidance had to be provided to make the class understand that this was not an assignment of the ordinary kind, i.e. they were not supposed to present a sequence of calculations but rather to attempt to make their new insight clear to each other. We can let this be

- Lesson 1a: Even when the format is radically different from well-known – and therefore "safe" – modes of learning, a tendency prevails among learners to attempt to squeeze this new mode into better known work patterns
- Lesson 1b: The author had in fact expected that something akin to the Micro Project was very familiar to the pupils from their primary school – but this may have been an ill-founded assumption

The Micro Project soon established itself as a useful way of regularly handing the initiative to the pupils, so when the author took over a Design-Technology class, an attempt was made to introduce vectors and their relations to other mathematical concept via cartography. A selection of mapping projections was made (Mercator's, equi-rectangular, Gall-Peters', sinusoidal, Kavrayskiy's, Aitoff's, Albers', Lambert's and stereographic projection), one given to each team; and the teams were invited to visualize the mechanism and illustrate and discuss the pros and cons of each of these. The necessary tools were made available to them and the work sheets provided links to more information. (This feature became regular when, in the early days, a pupil declared "I couldn't find anything about it on the Internet". This was quoted, along with the observation that this is equivalent to saying "I was late because the sun didn't rise at its usual time". Links were provided to English, German and French sources).

The cartography Micro Project was an abysmal failure.

To this day, the author refuses to blame the result on the Micro Project itself, especially since one team actually produced a fine presentation. – But the remaining teams either did not show up or 2 members appeared, claiming that the other 2 had the result of their work – or simply declared that they had not looked at the material. The author later learned that this particular class had a long history of absenteeism, lack of motivation and even a rebellious attitude to the effect that they didn't want to learn this, that or the other.

- Lesson 2a: Micro Projects are not miracle cures
- Lesson 2b: Before implementing a Micro Project make sure the learners are ready for it

And to this section on an optimistic note:

A large class (31 pupils) of 10^{th} graders were given a Micro Project on the subject of Extensions of the Number Concept. The sub-topics were

- Numeralia (including e.g. positional notation)
- Spoken words representing numbers (the author habitually asks whether "sytten hundrede og to og halvtreds" is a particularly logical rendering of 1752)

- Fractions, including continued fractions
- The elementary operations of arithmetic with a glance to e.g. the abacus and the *Pascaline*, an early calculating machine
- Algebraic numbers, up to and including the non-solvability of quantic equations in radicals (without proofs…)
- Binary arithmetic
- Matrices
- Complex numbers

It should be added that the worksheets explicitly demanded of the pupils that they identified and discussed at least two questions in addition to those mentioned in the description of the sub-topic.

The result was astonishing. Each group had fearlessly tackled its assignment and gave a presentation that belied the fact that these pupils had only just started their secondary education – some of them were 15 years of age, i.e. children, however much they disapproved of that label.

- Lesson 3a: If you ask the right question, all you must do next is step aside
- Lesson 3b: The amount of talent "out there" is overwhelming

Altogether, developing and facilitating Micro Projects in a school setting was a rewarding experience; but the reader should be warned that the amount of work is considerable.

V Whither Micro Projects?

It would seem reasonable to attempt to implement Micro Projects in a university setting, but the following issues must be taken into consideration:

- Classes in elementary mathematics are large, often consisting of more than 100 students
- The time allotted to the presentation of background theory of individual topics may be inadequate – for instance, the author currently teaches Oscillation Theory in the span of three weeks
- Although politicians and top managers may be enthusiastic and individual teachers willing to do the work, a peculiar resistance of the "not in my back yard"-type may be encountered from middle managers

As for the first of these, it is a resource question. So far, the author has only been able to rescue the idea of *Contextualization-Decontextualization-Recontextualization* – this choice of wording is taken from (Hansen. H.C. [9]) – by insisting that exercises should not be about "x and y" throughout, but should instead move to authentic models as fast as possible.

Examples: The Hydrological Cycle as source of linear systems (Rose, S. E [16]); Tuned Mass Dampers providing systems of differential equations ([1] Abdelraheem Farghaly, A); Annual variation of radon levels to be treated with least square methods (Arvela, H., O. Holmgren and P. Hänninen [4]); Traces cut by CNC-machines providing nonlinear equations (LIU Qiang*, LIU Huan, and YUAN Songmei [14]); and many others, even if many of these must be watered down.

The second bullet point above harkens back to the observation by Klamkin, op. cit.: We are forced to squeeze a large curriculum into a narrow time frame. It must therefore be emphasized:

The problem of time allocation is much exacerbated by the fact that we cannot expect the students to master the algebra of the solution processes needed to handle interesting challenges — in other words by The Mathematics Problem

As for the third, which was observed and discussed – but not analyzed or documented – in the EDU-IT project of 1999-2000 (reports are now scattered, but see (Hansen [10])), it is neither mathematical nor pedagogical in nature, yet has a considerable influence on education. It deserves further study, and the interested reader may want to consult (Askling, B and B. Stensaker [5]) or (Blackmore, P. and R. Blackwell [6])

There is still the possibility of introducing the Micro Project in 4th term courses or later, where classes are smaller and the curriculum somewhat more flexible. The present author hopes to be able to report on this in the not-too-distant future.

VI By way of conclusion

Without necessarily accepting the full eight-leaved *competency rose* of (Anonymous [2]), one may at least be allowed to wish for a balanced acquisition of

- Insight into the architecture of mathematics
- Competency in the handling of mathematical models
- Skills in the necessary calculations

Calculational skills can only be acquired by training via exercises. These are the mathematical equivalent of the pianist's scales and arpeggios. But no one would practice endlessly on c-minor scales in parallel sixths or the like without ever playing a real composition and at least hearing a recital or a concert. Likewise, an overdose of calculation without the least reference to either the physical reality or the bigger picture – and preferably both – leads to a Plato's Cave-like idea of what mathematics is about.

By focusing too much on these almost mechanical skills – which we cannot do without but which are not the essence of what we try to teach – we have become dispensers of sterile occupational therapy, merchants of death by *x* and *y*.

We need a lot of alternative ways of presenting our discipline. The Micro Project and the experiences it brought with it is but one of many possible such. Although a bit exaggerated, one may paraphrase Cato the Elder:

PRAETEREA CENSEO EXERCITIONEM ARITHMETICAM ESSE PRAETERMITTENDAM

and take it from there.

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